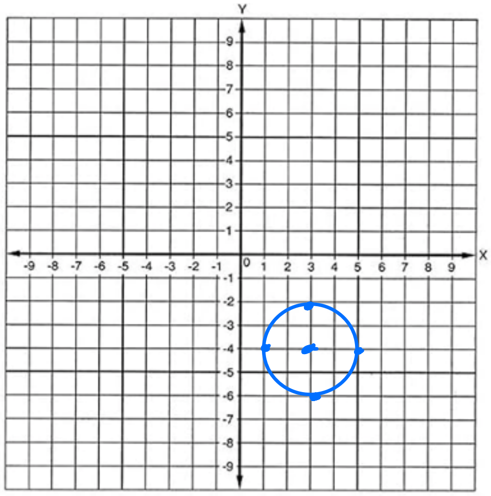
Period 4, Oct 24, 2024



$x^2-6x+y^2+8y+21=0$

$\qquad\qquad\qquad\qquad\quad -21 \quad -21$

Complete The Square

$1x^2-6x+9+y^2+8y+16=-21+9+16$

$(x-3)^2$

$a=1$

$b=8$

$\frac{b}{2}=\frac{8}{2}=4$

$\left(\frac{b}{2}\right)^2=(4)^2=16$

Center(3, -4) $\Longleftarrow (x-3)^2+(y+4)^2=4=2^2 \Leftarrow r^2$

Radius $=2$

Complete The Square Steps

1. $a=1$ ✓
2. Find $b \Rightarrow b=-6$ ✓
3. Find $\frac{b}{2}=\frac{-6}{2}=-3$ ✓
4. Find $\left(\frac{b}{2}\right)^2=(-3)^2=9$ ✓
5. add $\left(\frac{b}{2}\right)^2$ To both sides ✓
6. $\left(x+\frac{b}{2}\right)^2$ is The Square

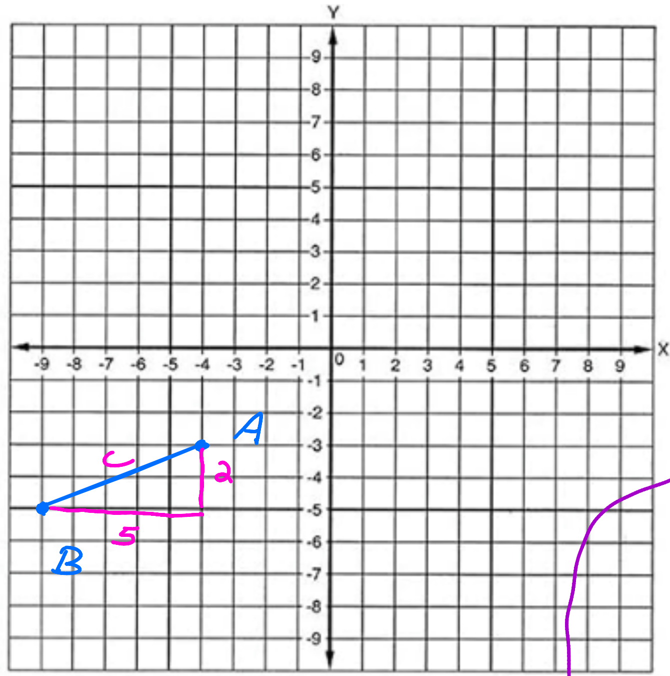
Center 3, -4

radius $=\sqrt{4}=2$

$(x-h)^2+(y-k)^2=r^2$

Center (h,k)

r = radius



A B
$(-4, -3)$, $(-9, -5)$
x_1, y_1 x_2, y_2

Find The distance between

$a^2 + b^2 = c^2$

$2^2 + 5^2 = c^2$

$4 + 25 = c^2$

$\sqrt{29} = \sqrt{c^2}$

$+\sqrt{29} = c$

$dis = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

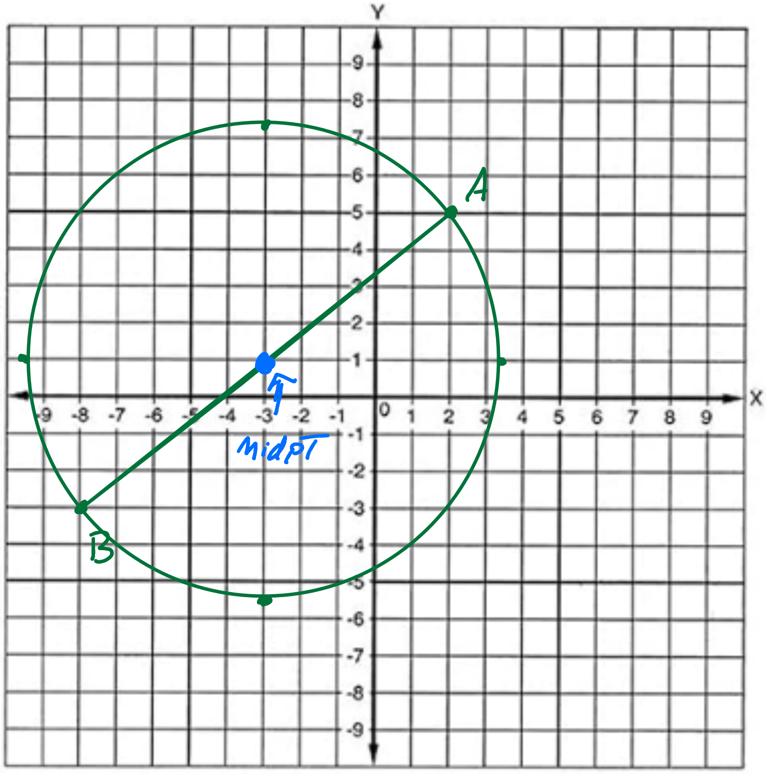
$= \sqrt{(-9 + +4)^2 + (-5 + +3)^2}$

$= \sqrt{(-5)^2 + (-2)^2} = \sqrt{25 + 4}$

$\sqrt{29}$

diameter $\overset{A}{(2,5)}, \overset{b}{(-8,-3)}$

$x_1\ y_1 \qquad x_2\ y_2$



MidPoint Formula

$\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$

$\left(\frac{2-8}{2}, \frac{5-3}{2}\right)$

$\left(\frac{-6}{2}, \frac{2}{2}\right) = (-3,1)$

center

radius

Find distance From center To $(2,5)$ $x_1\ y_1$

$(-3,1)$ $x_2\ y_2$

$dis = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$dis = \sqrt{(-3-2)^2+(1-5)^2}$

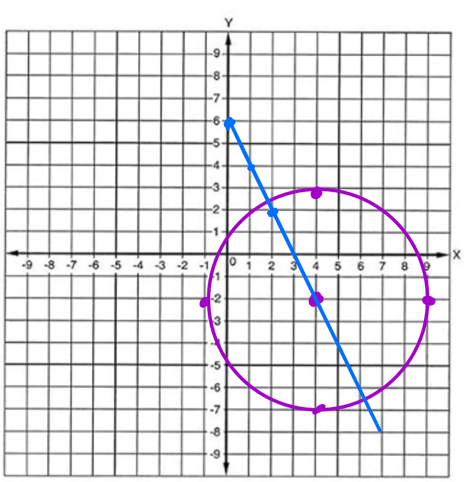
$= \sqrt{(-5)^2+(-4)^2} = \sqrt{25+16}$

radius $= \sqrt{41}$

Circle

$(x - {}^-3)^2 + (y-1)^2 = (\sqrt{41})^2$

$(x+3)^2+(y-1)^2 = 41$



Circle $(x-4)^2+(y+2)^2 = 25$

center $(4,-2)$, radius $= \sqrt{25} = 5$

Line $y = -2x+6$

where does The line cross The circle?

Take $y=-2x+6$ Plug into circle

$(x-4)^2+(-2x+6+2)^2 = 25 \Rightarrow (x-4)^2+(-2x+8)^2 = 25$

$x^2-8x+16+4x^2-32x+64 = 25 \Rightarrow 5x^2-40x+55=0$

$-25\ \ -25$

$a=5 \quad b=-40 \quad c=55$

The function $f(x) = (x+7)^5$ is one-to-one. $\Rightarrow \sqrt[5]{x} = \sqrt[5]{(y+7)^5}$ $\Rightarrow \sqrt[5]{x} = y + \cancel{7}$ $\Rightarrow \sqrt[5]{x} - 7 = y$

$-7 \quad -\cancel{7}$

a. Find an equation for $f^{-1}(x)$, the inverse function.

b. Verify that your equation is correct by showing that $f\left(f^{-1}(x)\right) = x$ and $f^{-1}(f(x)) = x$.

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a. Select the correct choice below and fill in the answer box(es) to complete your choice.

(Simplify your answer. Use integers or fractions for any numbers in the expression.)

- A. $f^{-1}(x) = \square$, for $x \neq \square$
- B. $f^{-1}(x) = \square$, for $x \geq \square$
- C. $f^{-1}(x) = \square$, for $x \leq \square$
- D. $f^{-1}(x) = \square$, for all x

$F(x) = (x+7)^5$

$F^{-1}(x) = \boxed{\sqrt[5]{x} - 7}$

$F(F^{-1}(x)) = (F^{-1}(x) + 7)^5 = (\sqrt[5]{x} \cancel{-7} \cancel{+7})^5 = (\sqrt[5]{x})^5 = x$

$F^{-1}(F(x)) = \sqrt[5]{F(x)} - 7 = \sqrt[5]{(x+7)^5} - 7$

$= x \cancel{+7} \cancel{-7} = x$

Given the function $f(x) = \sqrt[3]{x} + 7$, complete parts a through c.

$x = \sqrt[3]{y} + \cancel{7} \Rightarrow (x-7)^3 = (\sqrt[3]{y})^3$

$-7 \quad -\cancel{7}$

$(x-7)^3 = y$

(a) Find an equation for $f^{-1}(x)$.

(b) Graph f and f^{-1} in the same rectangular coordinate system.

(c) Use interval notation to give the domain and the range of f and f^{-1}.

(Hint: To solve for a variable involving an nth root, raise both sides of the equation to the nth power, $\left(\sqrt[n]{y}\right)^n = y$.)

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a) Find $f^{-1}(x)$. Select the correct choice below and fill in the answer box(es) to complete your choice.

(Simplify your answer. Use integers or fractions for any numbers in the expression.)

- A. $f^{-1}(x) = \square$, $x \neq \square$
- B. $f^{-1}(x) = \square$, for all x
- C. $f^{-1}(x) = \square$, $x \leq \square$
- D. $f^{-1}(x) = \square$, $x \geq \square$

Given the function $f(x) = (x - 11)^2$, $x \le 11$, complete parts a through c.

$y \ge 0$

(a) Find an equation for $f^{-1}(x)$.

(b) Graph f and f^{-1} in the same rectangular coordinate system.

(c) Use interval notation to give the domain and the range of f and f^{-1}.

x	$y=(x-11)^2$
11	0
10	$1 = (10-11)^2 = (-1)^2$
9	$4 = (9-11)^2 = (-2)^2$
8	$9 = (8-11)^2 = (-3)^2$

$\sqrt{x} = \sqrt{(y-11)^2}$

$\pm\sqrt{x} = y - 11$

$+11 \qquad +11$

$11 \pm \sqrt{x} = y$

$11 \pm \sqrt{4} = 9$

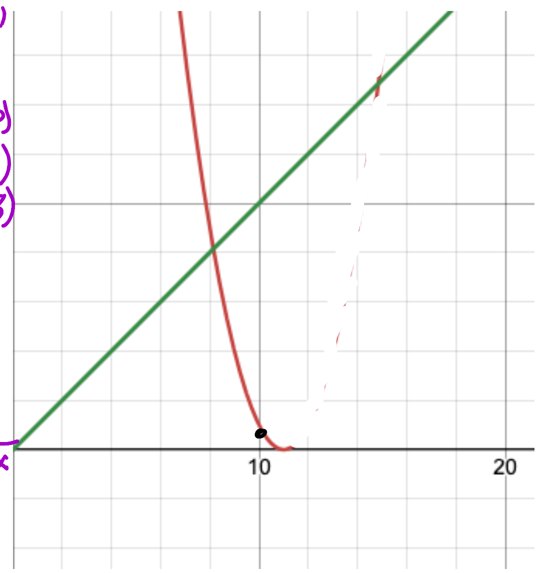
$11 \pm 2 = 9$

must be −

$11 - 2 = 9$

$F(x)$	$F^{-1}(x)$
(11, 0)	(0, 11)
(10, 1)	(1, 10)
(9, 4)	(4, 9)
(8, 9)	(9, 8)

$F^{-1}(x) = 11 - \sqrt{x}$



The function $f(x) = \dfrac{5x+9}{x-4}$ is one-to-one. $\qquad F^{-1}(x) = \dfrac{4x+9}{x-5}$

a. Find an equation for $f^{-1}(x)$, the inverse function.

b. Verify that your equation is correct by showing that $f\left(f^{-1}(x)\right) = x$ and $f^{-1}(f(x)) = x$.

$(y-4)x = \dfrac{5y+9}{\cancel{y-4}}\cancel{(y-4)} \Rightarrow xy - \cancel{4}x = 5y + 9 \Rightarrow xy - 5y = 4x + 9$

$-5y \; +4x \quad -5y \; +4x$

$y\cancel{(x-5)} = \dfrac{4x+9}{(x-5)}$, $\quad \cancel{(x-5)}$

$y = \left(\dfrac{4x+9}{x-5}\right)$

$F(F^{-1}(x)) = \dfrac{5F^{-1}(x) + 9}{F^{-1}(x) - 4} = \left[\dfrac{5\left(\dfrac{4x+9}{x-5}\right) + 9}{\dfrac{4x+9}{x-5} - 4}\right]$

$$\frac{\frac{20x+45}{x-5}+\frac{9(x-5)}{1(x-5)}}{\frac{4x+9}{x-5}-\frac{4(x-5)}{1(x-5)}} = \frac{\frac{20x+45}{x-5}+\frac{9x-45}{x-5}}{\frac{4x+9}{x-5}-\frac{4x-20}{x-5}} = \frac{\frac{20x+\cancel{45}+9x-\cancel{45}}{x-5}}{\frac{\cancel{4x}+9-\cancel{4x}+20}{x-5}}$$

$$\frac{\cancel{29}x}{\cancel{(x-5)}}\cdot\frac{\cancel{(x-5)}}{\cancel{29}} = \boxed{x}$$

Given the function $f(x) = \sqrt{x-9}$, complete parts a through c. $x \geq 9$

$\Rightarrow (x)^2 = (\sqrt{y-9})^2 \Rightarrow x^2 = y - 9 \Rightarrow x^2 + 9 = y$

$+9 \quad +9$

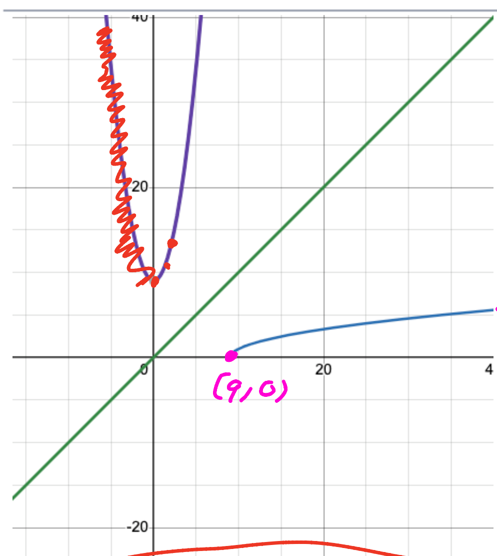
Range $y \geq 9$

(a) Find an equation for $f^{-1}(x)$.

(b) Graph f and f^{-1} in the same rectangular coordinate system.

(c) Use interval notation to give the domain and the range of f and f^{-1}.

(Hint: To solve for a variable involving an nth root, raise both sides of the equation to the nth power, $(\sqrt[n]{y})^n = y$.)



$F(x) = \sqrt{x-9}$

x	y
9	$0 = \sqrt{0} = \sqrt{9-9}$
10	$1 = \sqrt{1} = \sqrt{10-9}$
13	$2 = \sqrt{4} = \sqrt{13-9}$

$x \geq 0$

$F^{-1}(x) = x^2 + 9$

x	y
0	9
1	10
2	13

The function $f(x) = 7x + 6$ is one-to-one.

$x = 7y + 6 \Rightarrow x - 6 = 7y \Rightarrow \frac{x-6}{7} = y$

a. Find an equation for f^{-1}, the inverse function.

b. Verify that your equation is correct by showing that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

$F(F^{-1}(x)) = 7(F^{-1}(x)) + 6 \Rightarrow \cancel{7}\left(\frac{x-6}{\cancel{7}}\right) + 6 \Rightarrow F(F^{-1}(x)) = x \cancel{-6} \cancel{+6} = x$

$$\frac{(5-2i)(6-2i)}{(6+2i)(6-2i)} = \frac{30-10i-12i+4i^2}{36-\cancel{12i}+\cancel{12i}-4i^2} = \frac{30-22i+4(-1)}{36-4(-1)} = \frac{26-22i}{40}$$

30   -4

$36+4$

$$\boxed{\frac{\cancel{2}(13-11i)}{\cancel{2}\cdot 20}}$$

Use the graph of f to draw the graph of its inverse function.